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Nombre: _____

Carnet: _____ Sección: ___

**TERCER PARCIAL CO3121**

Justifique adecuadamente sus respuestas.

1. Las concentraciones de dos contaminantes atmosféricos en un día de verano en ciudad de México, medidas en miligramos por litro de aire, pueden modelarse como variables aleatorias $X$ e $Y$, cuya densidad conjunta está dada por

$$f(x,y) = \begin{cases} x^2 + \frac{1}{3}\,xy, & 0 \le x \le 1,\ 0 \le y \le 2 \\ 0 & \text{en otro caso.} \end{cases}$$

   (a) Hallar la probabilidad de que el contaminante $X$ tenga una concentración mayor a 0.6 milig/litro.
   (b) Hallar la densidad marginal de $Y$.
   (c) Cuando $X > 0.5$ y $Y > 1.5$ se debe decretar condición de alarma por contaminación atmosférica. ¿Cual es la probabilidad de tener que decretar dicha alarma en un día de verano?

2. Los índices de glicemia y grasa corporal estandarizados, en un adulto del Estado Vargas, pueden considerarse como un par de variables $X$ e $Y$ con densidad conjunta

$$f(x,y) = \begin{cases} 4x^2y + 2y^5 & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{en otro caso.} \end{cases}$$

   (a) Hallar la probabilidad de que, en un adulto de Vargas tomado al azar, el índice estandarizado de glicemia sea inferior a 0.5.
   (b) Hallar la densidad marginal del índice de grasa corporal.
   (c) ¿En que porcentaje de los individuos de la población adulta son ambos índices inferiores a 0.33?

3. Sea $X$ una variable aleatoria con distribución $\exp(\lambda)$.
   (a) Encuentre la densidad de probabilidad de la variable

$$Y = 1 - \mathrm{e}^{-\lambda X}.$$

   (b) ¿Cual es la función de probabilidad de $Z = \lfloor 10Y \rfloor + 1$?

4. Sea $X$ variable aleatoria con densidad $f(x) = n\,x^{n-1}$, para $x \in [0,1]$, siendo $n$ un natural positivo.
   (a) Hallar la densidad de probabilidad de la variable $Y = X^n$.
   (b) ¿Cual es la función de probabilidad de $D = \lfloor 6Y \rfloor + 1$?

Principales Distribuciones

| Distribución | $p(x)$ o $f(x)$ | Rango($X$) |
|---|---|---|
| Bin$(n,p)$ | $\binom{n}{x}p^x(1-p)^{n-x}$ | $\{0,1,2,\ldots,n\}$ |
| Geo$(p)$ | $(1-p)^{x-1}p$ | $\{0,1,2,\ldots\}$ |
| BinNeg$(k,p)$ | $\binom{x-1}{k-1}p^k(1-p)^{x-k}$ | $\{k,k+1,k+2,\ldots\}$ |
| Poisson$(\lambda)$ | $\mathrm{e}^{-\lambda}\frac{\lambda^x}{x!}$ | $\{0,1,2,\ldots\}$ |
| N$(\mu,\sigma^2)$ | $\frac{1}{\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ | $x\in\mathbb{R}$ |
| Unif$(a,b)$ | $\frac{1}{b-a}$ | $a<x<b$ |
| exp$(\lambda)$ | $\lambda\mathrm{e}^{-\lambda x}$ | $x>0$ |
| Gamma$(\alpha,\lambda)$ | $\frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}\mathrm{e}^{-\lambda x}$ | $x>0$ |

Función de Distribución Acumulativa N(0,1): $F(z)=\int_{-\infty}^{z}\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-u^2/2}\mathrm{d}u$

| x | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5 | 0.50398 | 0.50797 | 0.51196 | 0.51595 | 0.51993 | 0.52392 | 0.5279 | 0.53188 | 0.53585 |
| 0.1 | 0.53982 | 0.54379 | 0.54775 | 0.55171 | 0.55567 | 0.55961 | 0.56355 | 0.56749 | 0.57142 | 0.57534 |
| 0.2 | 0.57925 | 0.58316 | 0.58706 | 0.59095 | 0.59483 | 0.5987 | 0.60256 | 0.60641 | 0.61026 | 0.61409 |
| 0.3 | 0.61791 | 0.62171 | 0.62551 | 0.6293 | 0.63307 | 0.63683 | 0.64057 | 0.6443 | 0.64802 | 0.65173 |
| 0.4 | 0.65542 | 0.65909 | 0.66275 | 0.6664 | 0.67003 | 0.67364 | 0.67724 | 0.68082 | 0.68438 | 0.68793 |
| 0.5 | 0.69146 | 0.69497 | 0.69846 | 0.70194 | 0.7054 | 0.70884 | 0.71226 | 0.71566 | 0.71904 | 0.7224 |
| 0.6 | 0.72574 | 0.72906 | 0.73237 | 0.73565 | 0.73891 | 0.74215 | 0.74537 | 0.74857 | 0.75174 | 0.7549 |
| 0.7 | 0.75803 | 0.76114 | 0.76423 | 0.7673 | 0.77035 | 0.77337 | 0.77637 | 0.77935 | 0.7823 | 0.78523 |
| 0.8 | 0.78814 | 0.79102 | 0.79389 | 0.79673 | 0.79954 | 0.80233 | 0.8051 | 0.80784 | 0.81057 | 0.81326 |
| 0.9 | 0.81593 | 0.81858 | 0.82121 | 0.82381 | 0.82639 | 0.82894 | 0.83147 | 0.83397 | 0.83645 | 0.83891 |
| 1.0 | 0.84134 | 0.84375 | 0.84613 | 0.84849 | 0.85083 | 0.85314 | 0.85542 | 0.85769 | 0.85992 | 0.86214 |
| 1.1 | 0.86433 | 0.8665 | 0.86864 | 0.87076 | 0.87285 | 0.87492 | 0.87697 | 0.87899 | 0.88099 | 0.88297 |
| 1.2 | 0.88493 | 0.88686 | 0.88876 | 0.89065 | 0.89251 | 0.89435 | 0.89616 | 0.89795 | 0.89972 | 0.90147 |
| 1.3 | 0.90319 | 0.9049 | 0.90658 | 0.90824 | 0.90987 | 0.91149 | 0.91308 | 0.91465 | 0.9162 | 0.91773 |
| 1.4 | 0.91924 | 0.92073 | 0.92219 | 0.92364 | 0.92506 | 0.92647 | 0.92785 | 0.92921 | 0.93056 | 0.93188 |
| 1.5 | 0.93319 | 0.93447 | 0.93574 | 0.93699 | 0.93821 | 0.93942 | 0.94062 | 0.94179 | 0.94294 | 0.94408 |
| 1.6 | 0.9452 | 0.9463 | 0.94738 | 0.94844 | 0.94949 | 0.95052 | 0.95154 | 0.95254 | 0.95352 | 0.95448 |
| 1.7 | 0.95543 | 0.95636 | 0.95728 | 0.95818 | 0.95907 | 0.95994 | 0.96079 | 0.96163 | 0.96246 | 0.96327 |
| 1.8 | 0.96406 | 0.96485 | 0.96562 | 0.96637 | 0.96711 | 0.96784 | 0.96855 | 0.96925 | 0.96994 | 0.97062 |
| 1.9 | 0.97128 | 0.97193 | 0.97257 | 0.97319 | 0.97381 | 0.97441 | 0.975 | 0.97558 | 0.97614 | 0.9767 |
| 2.0 | 0.97724 | 0.97778 | 0.9783 | 0.97882 | 0.97932 | 0.97981 | 0.9803 | 0.98077 | 0.98123 | 0.98169 |
| 2.1 | 0.98213 | 0.98257 | 0.98299 | 0.98341 | 0.98382 | 0.98422 | 0.98461 | 0.98499 | 0.98537 | 0.98573 |
| 2.2 | 0.98609 | 0.98644 | 0.98679 | 0.98712 | 0.98745 | 0.98777 | 0.98808 | 0.98839 | 0.98869 | 0.98898 |
| 2.3 | 0.98927 | 0.98955 | 0.98982 | 0.99009 | 0.99035 | 0.99061 | 0.99086 | 0.9911 | 0.99134 | 0.99157 |
| 2.4 | 0.9918 | 0.99202 | 0.99223 | 0.99245 | 0.99265 | 0.99285 | 0.99305 | 0.99324 | 0.99343 | 0.99361 |
| 2.5 | 0.99379 | 0.99396 | 0.99413 | 0.99429 | 0.99445 | 0.99461 | 0.99476 | 0.99491 | 0.99505 | 0.9952 |
| 2.6 | 0.99533 | 0.99547 | 0.9956 | 0.99573 | 0.99585 | 0.99597 | 0.99609 | 0.9962 | 0.99631 | 0.99642 |
| 2.7 | 0.99653 | 0.99663 | 0.99673 | 0.99683 | 0.99692 | 0.99702 | 0.9971 | 0.99719 | 0.99728 | 0.99736 |
| 2.8 | 0.99744 | 0.99752 | 0.99759 | 0.99767 | 0.99774 | 0.99781 | 0.99788 | 0.99794 | 0.99801 | 0.99807 |
| 2.9 | 0.99813 | 0.99819 | 0.99824 | 0.9983 | 0.99835 | 0.99841 | 0.99846 | 0.99851 | 0.99855 | 0.9986 |
| 3.0 | 0.99865 | 0.99869 | 0.99873 | 0.99877 | 0.99881 | 0.99885 | 0.99889 | 0.99892 | 0.99896 | 0.99899 |
| 3.1 | 0.99903 | 0.99906 | 0.99909 | 0.99912 | 0.99915 | 0.99918 | 0.99921 | 0.99923 | 0.99926 | 0.99928 |
| 3.2 | 0.99931 | 0.99933 | 0.99935 | 0.99938 | 0.9994 | 0.99942 | 0.99944 | 0.99946 | 0.99948 | 0.99949 |
| 3.3 | 0.99951 | 0.99953 | 0.99954 | 0.99956 | 0.99958 | 0.99959 | 0.99961 | 0.99962 | 0.99963 | 0.99965 |
| 3.4 | 0.99966 | 0.99967 | 0.99968 | 0.99969 | 0.9997 | 0.99971 | 0.99972 | 0.99973 | 0.99974 | 0.99975 |